Grades 3-5 Mathematics Item Specification Claim 3	
This claim refers to a recurring theme in the CCSSM content and practice standards: the ability to construct and present a clear, logical, convincing argument. For older students this may take the form of a rigorous deductive proof based on clearly stated axioms. For younger students this will involve more informal justifications. Assessment tasks that address this claim will typically present a claim or a proposed solution to a problem and will ask students to provide, for example, a justification, an explanation, or counter-example. (*Mathematics Content Specifications, p.63*) Communicating mathematical reasoning is not just a requirement of the Standards for Mathematical Practice—it is also a recurrent theme in the Standards for Mathematical Content. For example, many content standards call for students to explain, justify, or illustrate.	
Primary Claim 3: Communicating Reasoning: Students clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.	
Secondary Claim(s): Items/tasks written primarily to assess Claim 3 will necessarily involve some Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 3 targets in the item form. If Claim 2 or Claim 4 targets are also directly related to the item/task, list those following the Claim 1 targets in order of prominence.	
Primary Content Domain: Each item/task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to and including the targeted grade within and across domains.	
Secondary Content Domain(s): While tasks developed to assess Claim 3 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate.	
DOK Levels	1, 2, 3
Allowable Response Types	**Response Types:** Multiple Choice, single correct response (MC); Multiple Choice, multiple correct response (MS); Equation/Numeric (EQ); Drag and Drop, Hot Spot, and Graphing (GI); Matching Table (MA); Fill-in Table (TI) No more than five choices in MS and MA items. Short Text–Performance tasks only **Scoring:** Scoring rules and answer choices will focus on a student's ability to solve problems and/or to apply appropriate strategies to solve problems. For some problems, multiple correct responses and/or strategies are possible. • MC and MS items will be scored as correct/incorrect (1 point) • If MA items require two skills, they will be scored as: o All correct choices (2 points); at least ½ but less than all correct choices (1 point)

	o Justification[1] for more than 1 point **must be** clear in the scoring rules o Where possible, include a "disqualifier" option that if selected would result in a score of 0 points, whether or not the student answered ½ correctly. • EQ, GI, and TI items will be scored as: o Single requirement items will be scored as correct/incorrect (1 point) o Multiple requirement items: All components correct (2 points); at least ½ but less than all correct (1 point) o Justification for more than 1 point **must be** clear in the scoring rules
Allowable Stimulus Materials	Effort must be made to minimize the reading load in problem situations. Use tables, diagrams with labels, and other strategies to lessen the reading load. Use simple subject-verb-object (SVO) sentences; use contexts that are familiar and relevant to students at the targeted grade level. Target-specific stimuli will be derived from the Claim 1 targets used in the problem situation. All real-world problem contexts will be relevant to the age of the students. Stimulus guidelines specific to task models are given below.
Construct Relevant Vocabulary	Refer to the Claim 1 specifications to determine Construct Relevant Vocabulary associated with specific content standards.
Allowable Tools	Any mathematical tools appropriate to the problem situation and the Claim 1 target(s). Some tools are identified in Standard for Mathematical Practice #5 and others can be found in the language of specific standards.
Target-Specific Attributes	CAT items should take from 2 to 5 minutes to solve; Claim 3 items that are part of a performance task may take 3 to 10 minutes to solve.
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[2] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear,

[1] For a CAT item to score multiple points, either distinct skills must be demonstrated that earn separate points or distinct levels of understanding of a complex skill must be tied directly to earning one or more points.

[2] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[3]
Development Notes	• Items and task assessing Claim 3 may involve application of more than one standard. The focus is on communicating reasoning rather than demonstrating mathematical concepts or simple applications of mathematical procedures. • Targeted content standards for Claim 3 should belong to the major work of the grade (reference table of standards shown below). • Claim 1 *Specifications* that cover the following standards should be used to help inform an item writer's understanding of the difference between how these standards are measured in Claim 1 versus Claim 3. Development notes have been added to many of the Claim 1 specifications that call out specific topics that should be assessed under Claim 3. • Claim 3 items that require any degree of hand scoring can only be developed for performance tasks for grades 3-5. At least 80% of the items written to Claim 3 should primarily assess the standards and clusters listed in the table that follows.

Grade 3	Grade 4	Grade 5
3.OA.B	4.OA.A.3	5.NBT.A.2
3.NF.A	4.NBT.A	5.NBT.B.6
3.NF.A.1	4.NBT.B.5	5.NBT.B.7
3.NF.A.2	4.NBT.B.6	5.NF.A.1
3.NF.A.3	4.NF.A	5.NF.A.2
3.MD.A	4.NF.A.1	5.NF.B
3.MD.C.7	4.NF.A.2	5.NF.B.3
	4.NF.B.3a	5.NF.B.4
	4.NF.B.3b	5.NF.B.7a
	4.NF.B.3c	5.NF.B.7b
	4.NF.B.4a	5.MD.C
	4.NF.B.4b	5.MD.C.5a
	4.NF.C	5.MD.C.5b
	4.NF.C.7	5.G.B*
		5.G.B.4*

*Denotes additional and supporting clusters

[3] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Assessment Targets: Any given item/task should provide evidence for several of the following assessment targets; each of the following targets should not lead to a separate task. Multiple targets should be listed in order of prominence as related to the item/task.
Target A: Test propositions or conjectures with specific examples. (DOK 2) Tasks used to assess this target should ask for specific examples to support or refute a proposition or conjecture (e.g., An item stem might begin, “Provide 3 examples to show why/how…”). **Target B: Construct, autonomously[4], chains of reasoning that will justify or refute propositions or conjectures[5]. (DOK 3, 4)** Tasks used to assess this target should ask students to develop a chain of reasoning to justify or refute a conjecture. Tasks for Target B might include the types of examples called for in Target A as part of this reasoning, but should do so with a lesser degree of scaffolding than tasks that assess Target A alone. Some tasks for this target will ask students to formulate and justify a conjecture. **Target C: State logical assumptions being used. (DOK 2, 3)** Tasks used to assess this target should ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate. **Target D: Use the technique of breaking an argument into cases. (DOK 2, 3)** Tasks used to assess this target should ask students to determine under what conditions an argument is true, to determine under what conditions an argument is not true, or both. **Target E: Distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in the argument—explain what it is. (DOK 2, 3, 4)** Tasks used to assess this target present students with one or more flawed arguments and ask students to choose which (if any) is correct, explain the flaws in reasoning, and/or correct flawed reasoning. **Target F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (DOK 2, 3)** In earlier grades, the desired student response might be in the form of concrete referents. In later grades, concrete referents will often support generalizations as part of the justification rather than constituting the entire expected response.

[4] By “autonomous” we mean that the student responds to a single prompt, without further guidance within the task.

[5] At the secondary level, these chains may take a successful student 10 minutes to construct and explain. Times will be somewhat shorter for younger students, but still giving them time to think and explain. For a minority of these tasks, subtasks may be constructed to facilitate entry and assess student progress towards expertise. Even for such “apprentice tasks” part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.

Grade 3 standards that lend themselves to communicating reasoning	**The following standards can be effectively used in various combinations in Grade 3 Claim 3 items:** **Operations and Algebraic Thinking (OA)** **3.OA.B:** Understand properties of multiplication and the relationship between multiplication and division. **Number and Operations—Fractions (NF)** **3.NF.A:** Develop understanding of fractions as numbers. **3.NF.A.1** Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*. **3.NF.A.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram. **3.NF.A.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **Measurement and Data (MD)** **3.MD.A:** Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. **3.MD.C:** Geometric measurement: understand concepts of area and relate area to multiplication and to addition. **3.MD.C.7** Relate area to the operations of multiplication and addition.

Grade 4 standards that lend themselves to communicating reasoning	**The following standards can be effectively used in various combinations in Grade 4 Claim 3 items:** **Operations and Algebraic Thinking (OA)** **4.OA.A.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Number and Operations in Base Ten (NBT)** **4.NBT.B:** Use place value understanding and properties of operations to perform multi-digit arithmetic **4.NBT.B5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **4.NBT.B.6** Find whole-number quotients and remainders with up to four-digit dividends and one-

Grade 4 standards that lend themselves to communicating reasoning	digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Number and Operations—Fractions (NF)** **4.NF.A:** Extend understanding of fraction equivalence and ordering. **4.NF.A.1** Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **4.NF.A.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **4.NF.B:** Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. **4.NF.B.3** Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. **a.** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. **b.** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* **c.** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. **4.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **a.** Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* **b.** Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* **4.NF.C:** Understand decimal notation for fractions, and compare decimal fractions. **4.NF.C.7** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Grade 5 standards that lend themselves to communicating reasoning	The following standards can be effectively used in various combinations in Grade 5 Claim 3 items:
	Number and Operations in Base Ten (NBT) **5.NBT.A.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **5.NBT.B.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NTB.B.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **Number and Operations—Fractions (NF)** **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **5.NF.A.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2*. **5.NF.B:** Apply and extend previous understandings of multiplication and division to multiply and divide fractions. **5.NF.B.3** Interpret a fraction as division of the numerator by the denominator (*a*/*b* = *a* ÷ *b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* **5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **5.NF.B.7** **a.** Interpret division of a unit fraction by a non-zero whole number and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

Grade 5 standards that lend themselves to communicating reasoning	**b.** Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* **Measurement and Data (MD)** **5.MD.C:** Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. **5.MD.C.5** **a.** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. **b.** Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. **Standards to integrate with the focus on fractions and whole number operations:** **Geometry (G)** **5.G.B:** Classify two-dimensional figures into categories based on their properties. **5.G.B.4** Classify two-dimensional figures in a hierarchy based on properties.

Range ALDs – Claim 3 Grades 3-5	**Level 1** Students should be able to base arguments on concrete referents such as objects, drawings, diagrams, and actions and identify obvious flawed arguments in familiar contexts.
	Level 2 Students should be able to find and identify the flaw in an argument by using examples or particular cases. Students should be able to break a familiar argument given in a highly scaffolded situation into cases to determine when the argument does or does not hold.
	Level 3 Students should be able to use stated assumptions, definitions, and previously established results and examples to test and support their reasoning or to identify, explain, and repair the flaw in an argument. Students should be able to break an argument into cases to determine when the argument does or does not hold.
	Level 4 Students should be able to use stated assumptions, definitions, and previously established results to support their reasoning or repair and explain the flaw in an argument. They should be able to construct a chain of logic to justify or refute a proposition or conjecture and to determine the conditions under which an argument does or does not apply.

Target 3A: Test propositions or conjectures with specific examples.

General Task Model Expectations for Target 3A

- Items for this target should focus on the core mathematical work that students are doing around numbers and operations, with mathematical content from other domains playing a supporting role in setting up the reasoning contexts.
- Items in this task model should probe the key mathematical structures that students at that grade-level are studying, such as the structure of base-ten numbers, fractions, or the four operations and their properties.
- In response to a claim or conjecture, the student should:
    - Find a counterexample if the claim is false,
    - Find examples and non-examples if the claim is sometimes true, or
    - Provide supporting examples for a claim that is always true without concluding that the examples establish that truth, unless there are only a finite number of cases and all of them are established one-by-one. The main role for using specific examples in this case is for students to develop a hypothesis that the conjecture or claim is true, setting students up for work described in Claim 3B.
- False or partially true claims that students are asked to find counterexamples for should frequently draw upon commonly held mathematical misconceptions.
- Note: Use appropriate mathematical language in asking students for a single example. While a single example can be used to refute a conjecture, it cannot be used to prove one is always true unless that is the one and only case.

Task Model 3A.1

- The student is presented with a proposition or conjecture and asked to give
    - A counterexample if the claim is false,
    - Examples and non-examples if the claim is sometimes true, or
    - One or more supporting examples for a claim that is always true without concluding that the examples establish that truth.

Example Item 3A.1a (Grade 3)
Primary Target 3A (Content Domain OA), Secondary Target 1D (CCSS 3.OA.B), Tertiary Target 3F

Marquis said, “The more numbers you multiply, the greater the product.” Then he wrote:

$2 \times 8 = 16$
$2 \times 5 \times 5 = 50$
$2 \times 3 \times 5 \times 2 = 60$

$60 > 50 > 16$

Give an example of a product of two numbers that is greater than $2 \times 5 \times 5$.

[ ] x [ ] > $(2 \times 5 \times 5)$

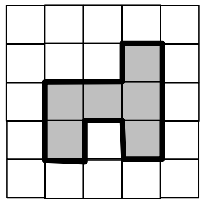
Enter the numbers in the two response boxes.

Rubric: (1 point) The student enters two numbers in the response boxes whose product is greater than 50. (e.g., 7 and 8).

Response Type: Equation/numeric

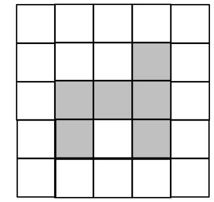
Example Item 3A.1b (Grade 4)
Primary Target 3A (Content Domain MD), Secondary Target 1I (CCSS 3.MD.D), Tertiary Target 3F

William shaded 6 squares in a grid to make the figure shown.

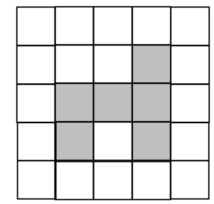


He claims that if he **adds 1 more** square to this figure in different places, the perimeter can be greater than, less than, or equal to the perimeter of the original figure.

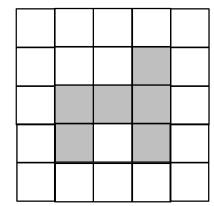
Part A. Click to shade one more square so the perimeter is greater than the original figure.



Part B. Click to shade one more square so the perimeter is less than the original figure.



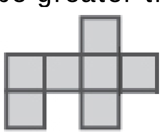
Part C. Click to shade one more square so the perimeter is equal to the original figure.



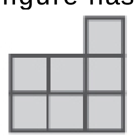
Rubric: (2 points) The student is able to provide an example that supports each conjecture.
(1 point) The student is able to provide two out of three correct examples.
(0 points) The student is unable to provide at least two correct examples.

Exemplar[6]:

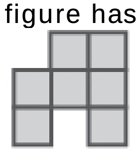
For Part A, the perimeter has to be greater than 14 units.



For Part B, the perimeter of the figure has to be less than 14 units.



For Part C, the perimeter of the figure has to be equal to 14 units.



Response Type: Hot Spot

[6] An exemplar is just one example of a correct response. Other correct responses are possible.

Example Item 3A.1c (Grade 5)
Primary Target 3A (Content Domain NBT), Secondary Target 1D (CCSS 4.NBT.B), Tertiary Target 3F

Nina says, “If you multiply a 2-digit number and a 1-digit number, you get a 3-digit number.”

Enter numbers in the table to give one example of when Nina’s claim is true, and another example that shows her claim is **not** always true.

Example of when –	**2-digit number**	**1-digit number**	**3-digit product**
Nina’s claim is true			
Nina’s claim is **not** true			

Rubric: (2 points) The student gives an example where the product is a three-digit number (e.g., $90 \times 2 = 180$) and an example where it is not (e.g., $10 \times 2 = 20$).
(1 point) The student gives an example where the product is a three-digit number or an example where it is not.

Response Type: Fill-in Table

Task Model 3A.2

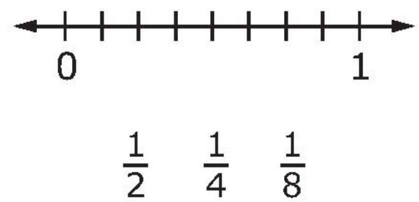
- The student is presented with one or more propositions or conjectures and several examples and asked implicitly or explicitly which examples support or refute each proposition.
- Items in this task model should cover all cases and not be unintentionally misleading about the truth status of a particular proposition or conjecture.

Example Item 3A.2a (Grade 3)
Primary Target 3A (Content Domain NF), Secondary Target 1F (CCSS 3.NF.3d), Tertiary Target 3F

Robert said, "When comparing two fractions with a numerator of 1, the fraction with the bigger denominator is always greater."

Part A
Drag each fraction to the correct location on the number line.

Part B
Is Robert's statement true? Click Yes or No.



Is Robert's statement true?

Click Yes or No.

Yes No

Interaction: The student drags fractions from the single-use palette to the number line and clicks on "Yes" or "No."

Rubric: (2 points) The student places all three fractions in the correct locations and answers "No."
(1 point) The student either places all the fractions in the correct locations and answers "Yes"; or places all fractions in the correct order but misses the correct location for one or more fractions and answers "No."

Response Type: Drag and Drop and Hot Spot

Example Item 3A.2b (Grade 4)
Primary Target 3A (Content Domain NBT), Secondary Target 1E (CCSS 4.NBT.B)

Click in the box that matches each division problem to the correct claim.

Claim	200 ÷ 5	777 ÷ 7	108 ÷ 9
When you divide a 3-digit number by a 1-digit number, the quotient can have **1 digit**.			
When you divide a 3-digit number by a 1-digit number, the quotient can have **2 digits**.			
When you divide a 3-digit number by a 1-digit number, the quotient can have **3 digits**.			

Rubric: (1 point) The student matches each quotient to the appropriate claim (e.g., Claim 2: 200 ÷ 5 and 108 ÷ 9. Claim 3: 777 ÷ 7.).

Response Type: Matching Table

Target 3B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.

General Task Model Expectations for Target 3B

- Items for this target should focus on the core mathematical work that students are doing around numbers and operations, with mathematical content from other domains playing a supporting role in setting up the reasoning contexts.
- Items for this target can probe a key mathematical structure such as the structure of base-ten numbers, fractions, or the four operations and their properties.
- Items for this target can require students to solve a multi-step, well-posed problem involving the application of mathematics to a real-world context. The difference between items for Claim 2A and Claim 3B is that the focus in 3B is on communicating the reasoning process in addition to getting the correct answer.
- Note that in grades 3–5, items can provide more structure than items for later grades to help them understand the expectations for justifying or refuting a proposition or conjecture.

Task Model 3B.1

- The student is presented with a proposition or conjecture. The student is asked to identify or construct reasoning that justifies or refutes the proposition or conjecture.
- Items in this task model often address more generalized reasoning about a class of problems or reasoning that generalizes beyond the given problem context even when it is presented in a particular case.

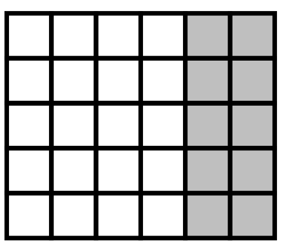
Example Item 3B.1a (Grade 3)

Primary Target 3B (Content Domain OA), Secondary Target 1B (CCSS 3.OA.B), Tertiary Target 3F

Bev said, "I can find 5×6 by adding 5×4 and 5×2."

She wrote this equation and drew this picture to show her thinking.

$5 \times 6 = 5 \times 4 + 5 \times 2$

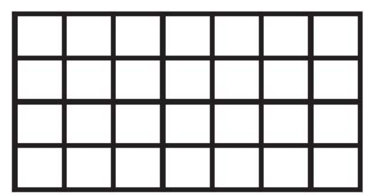


Mel wrote this equation: $4 \times 7 = 4 \times 3 + 4 \times 4$

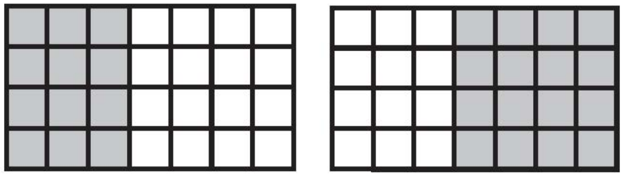
Is this equation true? Click on Yes or No.

Yes | No

Click on the squares to draw a picture that supports your answer.



Rubric: (1 point) The student identifies the equation as true and clicks to shade either a 4 x 3 rectangle or a 4 x 4 rectangle; see examples below.



Response Type: Hotspot

Example Item 3B.1b (Grade 4)
Primary Target 3B (Content Domain OA), Secondary Target 1B (CCSS 4.NBT.B), Tertiary Target 3F

Carter says, "8000 is 100 times as large as 80."

Choose three statements that support this claim.

Drag them into a logical order.

1.

2.

3.

So 8000 is 100 times as large as 80.

80 is 10 times as large as 8.
800 is 10 times as large as 80.
8000 is 10 times as large as 800.
$10 \times 10 = 100$
$10 \times 100 = 1000$
$80 \times 10 = 800$
$800 \times 10 = 8000$

Rubric: (1 point) The student selects three statements that complete an explanation for the claim and puts them in a logical order. In this particular example, the order doesn't matter.

Exemplars:

1. 800 is 10 times as big as 80.	1. $80 \times 10 = 800$
2. 8000 is 10 times as big as 800.	2. $800 \times 10 = 8000$
3. $10 \times 10 = 100$	3. $10 \times 10 = 100$

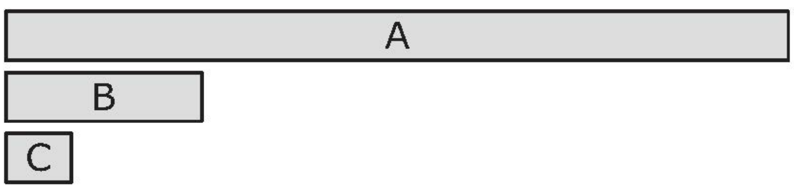
Response Type: Drag and Drop

Task Model 3B.2

- The student is asked a mathematical question and is asked to identify or construct reasoning that justifies his or her answer.
- Items in this task model often address more generalized reasoning about a class of problems or reasoning that generalizes beyond the given problem context even when it is presented in a particular case.

Example Item 3B.2a (Grade 4)
Primary Target 3B (Content Domain OA), Secondary Target 1B (CCSS), Tertiary Target 3F

Rectangle A is 4 times as long as rectangle B.
Rectangle B is 3 times as long as rectangle C.



How many times greater is rectangle A than rectangle C?

[ ] **times**

Choose three equations that, when taken together, support your claim. Drag them into a logical order.

1.

2.

3.

4 × A = B	3 × C = B
4 × B = A	4 × (3 × C) = A
3 × B = C	3 × (4 × C) = A

Rubric: (2 point) The student enters the correct multiplicative factor in the response box (e.g., 12) and selects three statements that support the claim and puts them in a logical order.
(1 point) The student does one or the other.

Exemplars:

1. 4 x B = A	1. 3 x C = B
2. 3 x C = B	2. 4 x B = A
3. 4 x (3 x C) = C	3. 4 x (3 x B) = A

Response Type: Equation/Numeric and Drag and Drop

Note: Functionality to combine these items types doesn't currently exist. The item could be implemented as a 1 point item if the scale factor is given.

Example Item 3B.2b (Grade 5)
Primary Target 3B (Content Domain MD), Secondary Target 1I (CCSS 5.MD.5), Tertiary Target 3F

The dimensions of a right rectangular prism are:

- length = 9 centimeters
- width = 3 centimeters
- height = 5 centimeters

What will happen to the volume of the right rectangular prism if the length, the width, and the height are each doubled?

The new volume will be [drop-down choices: 2, 4, 6, 8] times the original volume because $(2 \times 9)(2 \times 3)(2 \times 5) =$ [drop-down choices: 2, 4, 6, 8] $\times (9 \times 3 \times 5)$.

Rubric: (1 point) The student selects the correct multiplier (e.g., 8) in both drop-down menus.

Response Type: Drop-down menu

Note: Functionality for this item doesn't currently exist, though we anticipate to be able to offer drop-down items by 2018. The item could be implemented as a multiple choice in the meantime.

Task Model 3B.3

- Items for this target require the student to solve a multi-step, well-posed problem involving the application of mathematics to a real-world context.
- The difference between Claim 2 task models and this task model is that the student needs to provide some evidence of his/her reasoning. The difference between Claim 4 task models and this task model is that the problem is completely well posed and no extraneous information is given.

Example Item 3B.3a (Grade 3)
Primary Target 3B (Content Domain OA), Secondary Target 1D (CCSS 3.OA.D)

A bird ate 400 grams of food in 3 days. The bird ate 120 grams of food on Day 1, 150 grams of food on Day 2, and *g* grams of food on Day 3.

Day	Grams of Food
1	120
2	150
3	*g*

How many grams of food did the bird eat on Day 3? Enter your answer in the first response box.

In the second response box, enter an equation that you could solve to find the amount of food the bird ate on Day 3.

Rubric: (2 points) The student enters the correct number of grams of food on Day 3 and enters a correct (e.g., 130; $400 - 120 - 150 = x$, $120 + 150 + x = 400$, or equivalent equation).
(1 point) The student enters the correct number of grams of food on Day 3 or enters a correct equation.

Response Type: Equation/Numeric (2 response boxes)

Example Item 3B.3b (Grade 4)
Primary Target 3B (Content Domain MD), Secondary Target 1G (CCSS 4.MD.A)

- There are 60 seconds in a minute.
- There are 60 minutes in an hour.
- There are 24 hours in a day.

What is the total number of minutes in 1 day? Enter your answer in the first response box.

Write an expression that shows how you found your answer. Enter your expression in the second response box.

Rubric: (2 points) The student enters the correct number of minutes in a day in the first response box (1440) and a correct equation in the second response box (e.g., 60×24, 144×10, or equivalent expressions).
(1 point) The student enters the correct number of minutes in a day in the first response box or a correct equation in the second response box.

Response Type: Equation/Numeric (2 response boxes)

Target 3C: State logical assumptions being used.

General Task Model Expectations for Target 3C

- Items for this target should focus on the core mathematical work that students are doing around numbers and operations, with mathematical content from other domains playing a supporting role in setting up the reasoning contexts.
- For some items, the student must explicitly identify assumptions that
  - Make a problem well-posed, or
  - Make a particular solution method viable.
- When possible, items in this target should focus on assumptions that are commonly made implicitly and can cause confusion when left implicit.
- For some items, the student will be given a definition and be asked to reason from that definition.

Task Model 3C.1

- The student is asked to identify an unstated assumption that would make the problem well-posed or allow them to solve a problem using a given method.

Example Item 3C.1a (Grade 3)

Primary Target 3C (Content Domain OA), Secondary Target 1B (CCSS 3.OA.B)

A 20 meter rope is cut into 4 pieces. Jenny says you can find the length of each piece by finding 20 ÷ 4.

What statement best describes Jenny's claim?

A. Jenny's claim is false. She should add 4 and 20 instead.
B. Jenny's claim is false. She should multiply 4 and 20 instead.
C. Jenny's claim is true if you assume that each piece is 4 meters long.
D. Jenny's claim is true if you assume that the pieces are all equal in length.

Rubric: (1 point) The student selects the correct statement (e.g., D).

Response Type: Multiple Choice, single correct response

Example Item 3C.1b (Grade 5)
Primary Target 3C (Content Domain OA), Secondary Target 1A (CCSS 4.OA.A)

Gil and Nina are comparing the numbers 3 and 12.

Gil says, “12 is 9 more than 3.”

Nina says, “12 is 4 times more than 3.”

What is true about Gil and Nina’s statements?

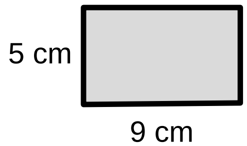
A. Nina is correct and Gil is not. You should multiply to compare the numbers.
B. Gil is correct and Nina is not. You should add to compare the numbers.
C. They are both correct. They just compared using different operations.
D. Neither one is correct. You have to compare like this: 12 > 3.

Rubric: (1 point) The student selects the correct statement (e.g., C).

Response Type: Multiple Choice, single correct response

Example Item 3C.1c (Grade 5)
Primary Target 3C (Content Domain G, MD), Secondary Target 1K (CCSS 5.G.B, 4.MD.A.3), Tertiary Target 3D

Carrie saw the figure below and said that its area is $5 \times 9 = 45$ square centimeters.



Which statement best supports Carrie’s claim?

A. It is true if the opposite sides have the same length.
B. It is true if the figure is a rectangle.
C. It is false if the opposite sides have the same length.
D. It is false if the figure is a rectangle.

Rubric: (1 point) The student selects the correct statement (e.g., B).

Response Type: Multiple Choice, single correct response

Example Item 3C.1d (Grade 5)
Primary Target 3C (Content Domain NF), Secondary Target 1F (CCSS 4.NF.A.2), Tertiary Target 3D

Flo ate $\frac{3}{4}$ of a sandwich and Arnie ate $\frac{2}{3}$ of a sandwich. If Arnie ate more, what must be true?

A. Flo's sandwich is bigger.
B. Arnie's sandwich is bigger.
C. The sandwiches are the same size.
D. It doesn't matter which sandwich is bigger.

Rubric: (1 point) The student selects the correct assumption (e.g., B).

Response Type: Multiple Choice, single correct response

Task Model 3C.2

- The student will be given one or more definitions or assumptions and be asked to reason from that set of definitions and assumptions.

Example Item 3C.2a (Grade 5)
Primary Target 3C (Content Domain G), Secondary Target 1K (CCSS 5.G.B)

Patrick is learning about quadrilaterals. He was given the following true statements.

- Opposite sides of all parallelograms have the same length.
- Opposite sides of all rectangles have the same length.
- All sides of a square have the same length.
- All rectangles are parallelograms.
- All rectangles have right angles.
- All squares have right angles.

Based on this information, Patrick assumes the following statements are always true. Which statement is **not** supported by the given information?

A. All squares are rectangles.
B. All squares are parallelograms.
C. All parallelograms are rectangles.
D. All parallelograms are quadrilaterals.

Rubric: (1 point) The student selects the correct response (e.g., C).

Response Type: Multiple choice, single correct response

Target 3D: Use the technique of breaking an argument into cases.

General Task Model Expectations for Target 3D

- Items for this target should focus on the core mathematical work that students are doing around numbers and operations, with mathematical content from other domains playing a supporting role in setting up the reasoning contexts.
- The student is given
  - A problem that has a finite number of possible solutions, some of which work and some of which don't, or
  - A proposition that is true in some cases but not others.
- Items for Claim 3 Target D should either present an exhaustive set of cases to consider or expect students to consider all possible cases in turn in order to distinguish it from items in other targets.
- In grades 3-5, the student will be given the cases to consider.

Task Model 3D.1

- The student is given a problem that has a finite number of possible solutions, some of which work and some of which don't.

Example Item 3D.1a (Grade 3)

Primary Target 3D (Content Domain OA), Secondary Target 1A (CCSS 3.OA.A)

Select **all** the ways can you divide 15 children into equal groups with none left over.

A. 2 groups
B. 3 groups
C. 4 groups
D. 5 groups

Rubric: (1 point) The student selects the possible number of groups (B and D).

Response Type: Multiple Choice, multiple select response

Example Item 3D.1b (Grade 4)
Primary Target 3D (Content Domain MD), Secondary Target 1K (CCSS 4.MD.C)

When you cut an obtuse angle into two smaller angles, what can be true? (Select **all** that apply.)

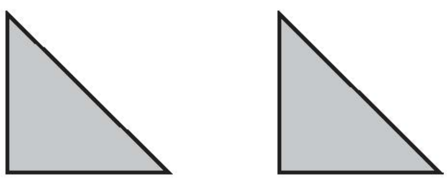
A. The two smaller angles can be less than 90 degrees.
B. At least one of the two smaller angles can be greater than 90 degrees.
C. Both of the two smaller angles can be greater than 90 degrees.

Rubric: (1 point) The student selects the possible cases (A and B).

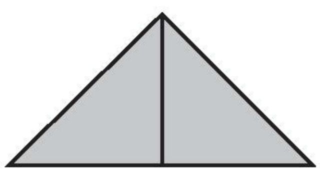
Response Type: Multiple Choice, multiple correct response

Example Item 3D.1c (Grade 5)
Primary Target 3D (Content Domain G), Secondary Target 1K (CCSS 5.G.B)

Nora has drawn two identical isosceles right triangles.



Here is a way to put them together so that they share a side and make another triangle.



Select **all** the quadrilaterals Nora can make with these triangles if she puts them together so that they share a side.

A. A square
B. A rectangle that is not a square
C. A rhombus that is not a square
D. A parallelogram that is not a rectangle

Rubric: (1 point) The student selects the possible cases (A and D).

Response Type: Multiple Choice, multiple select response

Task Model 3D.2

- The student is given a proposition and an exhaustive list of cases and asked to determine in which of those cases the proposition is true.

Example Item 3D.2a (Grade 3)
Primary Target 3D (Content Domain OA), Secondary Target 1B (CCSS 3.OA.B), Tertiary Target 3C

n is a whole number and $n \times 5 = 5$.

Identify which values of *n* make this equation true.

	True	False
When $n = 0$		
When $n = 1$		
When $n > 1$		
This is never true		

Rubric: (1 point) The student identifies the correct values of *n* (F, T, F, F)

Response Type: Matching Table

Example Item 3D.2b (Grade 4)
Primary Target 3D (Content Domain NF), Secondary Target 1G (CCSS 4.NF.A), Tertiary Target 3C

What must be true about *d* to make this inequality true?

$$\frac{3}{d} \geq \frac{3}{10}$$

Identify which values of *d* make this equation true.

	True	False
$d < 10$		
$d = 10$		
$d > 10$		

Rubric: (1 point) The student identifies the correct values of *d* (T, T, F)

Response Type: Matching Table

Example Item 3D.2c (Grade 5)
Primary Target 3D (Content Domain NF), Secondary Target 1? (CCSS 5.NF.B), Tertiary Target 3C

32×45 is greater than both 32 and 45. When is $a \times b$ between a and b?

Select **all** that apply.

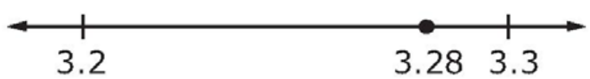
A. When $a > 1$ and $b > 1$
B. When $a < 1$ and $b > 1$
C. When $b < 1$ and $a > 1$
D. When $a < 1$ and $b < 1$

Rubric: (1 point) The student selects B and C.

Response Type: Multiple Choice, multiple correct response

Example Item 3D.2d (Grade 5)
Primary Target 3C (Content Domain NBT), Secondary Target 1C (CCSS 5.NBT.A), Tertiary Target 3F

Jenny says, "To round a decimal *d* between 3.2 and 3.3 to the nearest tenth, you just see which tenth it is closest to on the number line. For example, 3.28 is closer to 3.3 than 3.2, so it rounds to 3.3."



In which cases will Jenny's method work? (Select **all** that apply.)

A. Case 1: $3.25 < d \leq 3.3$
B. Case 2: $d = 3.25$
C. Case 3: $3.2 \leq d < 3.25$
D. Jenny's method doesn't usually doesn't work—it just worked for this example.

Rubric: (1 point) The student selects the correct cases (A and C).

Response Type: Multiple Choice, multiple correct response

Target 3E: Distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in the argument—explain what it is.

General Task Model Expectations for Target 3E

- Items for this target should focus on the core mathematical work that students are doing around numbers and operations, with mathematical content from other domains playing a supporting role in setting up the reasoning contexts.
- The student is presented with valid or invalid reasoning and told it is flawed or asked to determine its validity. If the reasoning is flawed, the student identifies, explains, and/or corrects the error or flaw.
- The error should be more than just a computational error or an error in counting, and should reflect an actual error in reasoning.
- Analyzing faulty algorithms is acceptable so long as the algorithm is internally consistent and it isn't just a mechanical mistake executing a standard algorithm.

Task Model 3E.1

- Some flawed reasoning or student work is presented and the student identifies and/or corrects the error or flaw.
- The student is presented with valid or invalid reasoning and asked to determine its validity. If the reasoning is flawed, the student will explain or correct the flaw.

Example Item 3E.1a (Grade 3)
Primary Target 3E (Content Domain OA), Secondary Target 1A (CCSS 3.OA.A), Tertiary Target 3C

Tasha is solving this problem:

> There 4 tanks with 10 fish in each tank. How many fish are there all together?

Tasha claims, "There are 4 + 10 = 14 fish all together."

Which statement best describes Tasha's claim?

A. Tasha correctly added to find the total.
B. Tasha should subtract instead.
C. Tasha should multiply instead.
D. Tasha should divide instead.

Rubric: (1 point) The student selects the correct statement (C).

Response Type: Multiple Choice, single correct response

Example Item 3E.1b (Grade 4)
Primary Target 3E (Content Domain NBT), Secondary Target 1E (CCSS 3.NBT.B)

Harvey was solving this problem:

There are 12 packets of gum each with a mass of 65 grams. What is the mass of all of the packets combined?

Harvey said, "I can multiply the tens places and the ones places and add them."

Then he wrote:

12 = 10 + 2
65 = 60 + 5
600 + 10 = 610
The total mass is 610 grams.

Which statement best describes Harvey's claim?

A. Harvey solved the problem correctly and got the right answer.
B. Harvey made a mistake in solving the problem but got the right answer anyway.
C. Harvey had a correct way of solving the problem but got the wrong answer.
D. Harvey's solution is not correct because he did not multiply the tens with the ones.

Rubric: (1 point) The student selects the correct statement (e.g., D).

Response Type: Multiple Choice, single correct response

Example Item 3E.1c (Grade 5)
Primary Target 3E (Content Domain NF), Secondary Target 1E (CCSS 5.NF.A)

Brian is adding $\frac{2}{3}$ + $\frac{7}{5}$. He wrote: $\frac{2}{3}+\frac{7}{5}=\frac{2+7}{3+5}=\frac{9}{8}$

Brian's approach is **not** correct. Select **all** of the statements that could indicate mistakes with Brian's approach.

A. He added the denominators.
B. He didn't write $\frac{7}{5}$ as a mixed number.
C. He didn't write his answer as a mixed number.
D. He added the numerators when the denominators were different.

Rubric: (1 point) The student clicks on the mistakes in the algorithm (A and S).

Response Type: Multiple Choice, multiple correct response

Task Model 3E.2

- Two or more approaches or chains of reasoning are given and the student is asked to identify the correct method and justification OR identify the incorrect method/reasoning and the justification.

Example Item 3E.2a (Grade 4)

Primary Target 3E (Content Domain NBT), Secondary Target 1E (CCSS 4.NBT.A), Tertiary Target 3C, Quaternary Target 3F

Zach and Nate both rounded 6481, but used different methods.

Zach thought about it this way:	Nate thought about it this way:
6481 rounds to 6480 6480 rounds to 6500 6500 rounds to 7000 So 6481 rounds to 7000.	6481 is closer to 6000 than to 7000, so it rounds to 6000.

Which statement best describes these methods?

A. Zach's method is correct.
B. Nate's method is correct.
C. Both methods are correct.
D. Neither method is correct.

Rubric: (1 point) The student selects the correct method (B).

Response Type: Multiple Choice, single correct response

Example Item 3E.2a (Grade 5)
Primary Target 3E (Content Domain NBT), Secondary Target 1E (CCSS 4.NBT.A), Tertiary Target 3C

Mr. Spivak's class was finding the volume of a right rectangular prism with dimensions 20 cm, 45 cm, and 80 cm.

Brigit said, "I tried two ways of multiplying the dimensions and got different answers. I can't figure out what went wrong."

She explained her two ways to Mr. Spivak.

First method:	**Second method:**
Step 1: I distributed.	Step 1: I broke apart the numbers.
$20 \times (45 \times 80) = (20 \times 45) + (20 \times 80)$	$20 \times 45 \times 80 = (2 \times 10) \times (5 \times 9) \text{ x } (8 \times 10)$
Step 2: I multiplied 20 by 45 and 20 by 80.	Step 2: I rearranged the numbers.
$= 900 + 1600$	
Step 3: Then I added.	Step 3: Then I multiplied everything.
$= 2500$	$= 72 \times (10 \times 100) = 72{,}000$

Which method has an error? Which step has the first error in that method?

Brigit's [drop-down options: first, second] method has an error. She made the error in step [drop-down options: 1, 2, 3].

Rubric: (1 point) The student selects the incorrect method (first) and identifies the step in which the error occurred (1).

Response Type: Drop-down Menu[7]

[7] This response is not yet supported by the Smarter Balanced item authoring tool, but is expected as an enhancement by 2017.

Target 3F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions

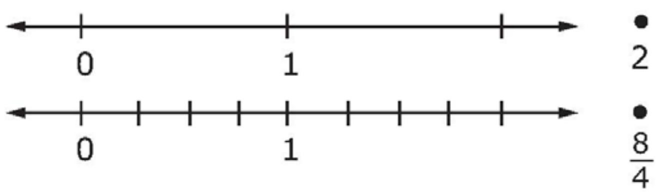
Task Model 3F.1

- The student uses concrete referents to help justify or refute an argument.
- Items in this task model should address content in standards that specifically call for number lines, diagrams, and contexts to be used as a basis for reasoning.

Example Item 3F.1a (Grade 3)
Primary Target 3F (Content Domain NBT), Secondary Target 1F (CCSS 3.NF.A), Tertiary Target 3B

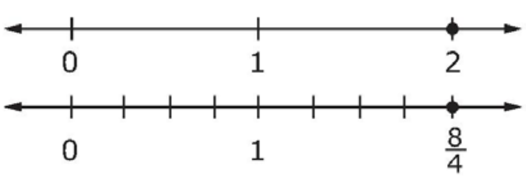
Compare $\frac{8}{4}$ and 2.

Part A
Plot each number on a number line.



Part B
$\frac{8}{4}$ [drop-down choices: <, =, >] 2

Rubric: (1 point) The student plots the points correctly (see below) and selects the correct comparison (=).



Response Type: Drop-down Menu, Graphing

Note: Functionality for this item type does not currently exist.

Example Item 3B.1b (Grade 3)
Primary Target 3F (Content Domain NF), Secondary Target 1F (CCSS 3.NF.A), Tertiary Target 3B

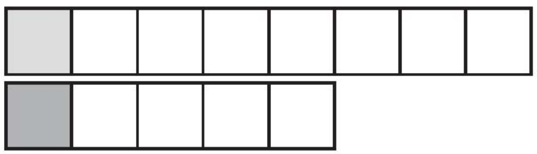
Part A
Which comparison between $\frac{1}{5}$ and $\frac{1}{8}$ is correct?

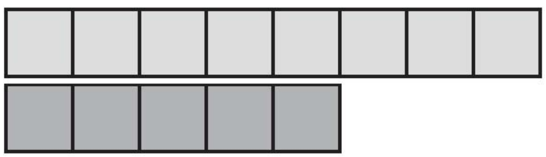
A. $\frac{1}{5} < \frac{1}{8}$

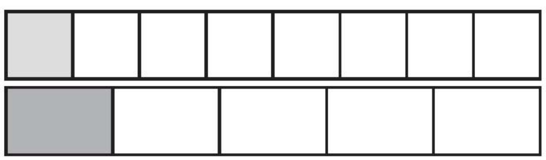
B. $\frac{1}{5} > \frac{1}{8}$

C. $\frac{1}{5} = \frac{1}{8}$

Part B
Choose a picture that supports your answer in *Part A*.

D.


E.


F.


Rubric: (1 point) The student selects the correct comparison and the correct picture (B, F).

Response Type: Drop-down Menu and Multiple Choice, single correct response

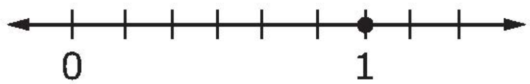
Example Item 3F.1c (Grade 4)

Primary Target 3F (Content Domain NBT), Secondary Target 1F (CCSS 4.NF.A), Tertiary Target 3B

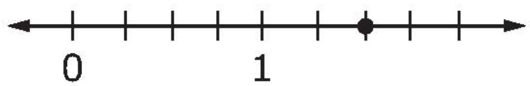
Which number line shows that $\frac{3}{4} = \frac{6}{8}$?

A.

B.



C.



D.

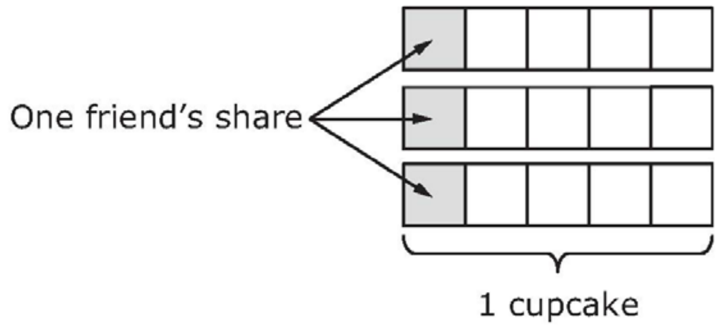
Rubric: (1 point) The student selects the correct number line (A).

Response Type: Multiple Choice, single correct response

Example Item 3F.1d (Grade 5)
Primary Target 3F (Content Domain NBT), Secondary Target 1F (CCSS 5.NF.B), Tertiary Target 3B

Complete the story about friends sharing cupcakes to show that $3 \div 5 = \frac{3}{5}$.

- 5 friends were sharing 3 cupcakes. They divided each cupcake into 5 equal pieces.
- Each piece is [drop-down menu choices: $\frac{1}{3}$, $\frac{1}{5}$, $\frac{3}{5}$] of a cupcake.
- Each friend got 1 piece of each cupcake.
- Each friend got [drop-down menu choices: $\frac{1}{3}$, $\frac{1}{5}$, $\frac{3}{5}$] of a cupcake in total.



Rubric: (1 point) The student selects the correct unit fraction ($\frac{1}{5}$) and the correct total amount each friend receives ($\frac{3}{5}$).

Response Type: Drop-down Menu