Strategies to Support English Learners in Mathematics

Why do our students need to communicate when learning mathematics?

Strategies to support student sense-making and communication . . .

- **5 Practices for Orchestrating Productive Mathematics Discussions (see back side)**
  Teachers design conversations and build connections between mathematical content.

- ***Language Scoop***
  - *Teachers listen for, highlight, and extend student language about mathematics.*
  - *Teachers' record language that is heard within student groups, teachers share out the language students used to describe their thinking and display them for all students to see.*
  - *Students are able to see and make connections between the oral and written representation of their communication. A class conversation is held about other ways/words that can be used to express these ideas.*

- ***Talk Moves***
  - *Revoicing – So you're saying . . .*
  - *Repeating – Can you repeat what she said in your own words?*
  - *Reasoning – Do you agree or disagree? Why does this make sense?*
  - *Adding On – Would someone like to add on to this?*
  - *Wait Time – Take your time . . .*
  - *Turn-and-Talk – Turn and talk to your neighbor . . .*
  - *Revise – Would you like to revise your thinking?*

- ***Notice/Wonder***
  - *What do you notice?*
  - *What do you wonder?*

- ***Read and Flip for Math Stories (word problems)***
  - *3 read protocol – Teachers create a focus for each of the reads. See sample below.*
  - *1st Read – The class choral reads, students retell the story, and the teacher charts student ideas.*
  - *2nd Read – The teacher reads (models a fluency read), students clarify ideas and add to what they know about the problem. Students write an answer statement.*
  - *3rd Read – Students read independently and draw a quick sketch.*

5 Practices for Orchestrating Productive Mathematics Discussions

Margaret S. Smith & Mary Kay Stein, NCTM & Corwin Press, 2011 (www.nctm.org)

0. **Selecting the Task**
    - Choose a task that promotes the mathematics you intend for students to learn.
    - The task should demand engagement with concepts and that stimulate students to make connections.

1. **Anticipating**
    - Do the problem yourself.
    - What are students likely to produce?
    - Which problems will be most likely be most useful in addressing the mathematics?

2. **Monitoring**
    - Listen, observe, and identify key strategies.
    - Keep track of approaches.
    - Pose questions to get students back on track or to think more deeply.

3. **Selecting**
    - This step is critical: What is the mathematics that you want to highlight?
    - Purposefully select student work that will advance mathematical ideas.

4. **Sequencing**
    - In what order do you want to present the student work samples?
    - Do you begin with the most common? The most accessible? The misconceptions?
    - How will students share their work? Board? Document Camera?

5. **Connecting**
    - Craft questions to make the mathematics visible.
    - Compare and contrast 2 or 3 students' work – What are the mathematical relationships?
    - What do parts of the students' work represent in the original problem? The solution? Other work done in the past?

Collaborative		Interpretive		Productive
Students construct and share arguments using concrete referents such as objects, drawings, diagrams, and actions. **MP.3**		Student listens actively as others share their strategies		Students try to communicate precisely to others, using clear definitions in discussion with others and in their own reasoning. **MP.6**
Students interact with others in written English about math concepts and ideas in a variety of ways (print, communicative technology, and multimedia). **MP.3**		Students explain correspondences between equations, verbal descriptions, tables and graphs. **MP.1**		Students write mathematical texts to present, describe, and explain ideas and information, using appropriate technology. **MP.5**
Students justify their thinking when creating a mathematical argument. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. **MP.3**		Students make sense of a real-world or mathematical word problem		Students support own opinions and evaluating other's opinions in speaking and writing in mathematics. **MP.3**
Students consider their language choices based on the various contexts (based on task, purpose, audience, and type of text). They think about and select precise mathematical vocabulary. **MP.3, 6**		Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **MP.3**		Students select and apply precise vocabulary and language structures to effectively convey mathematical ideas. **MP.6**
Students listen to and understand the approaches of others. They negotiate meaning and discuss a variety of ideas to support their ideas or conclusions. **MP.3**		Students are able to Identify important quantities in a practical situations and map their relationships using such tools as diagrams, two-way table, graphs, flow-chart, and formulas **MP.4**		Students write an addition equation to describe a situation, apply proportional reasoning to plan a school event or analyze a problem in a community. **MP.4**
Students try to use clear definitions in discussion with others. They state the meaning of their symbols and use assumptions, definitions, and previously established results in constructing arguments. **MP.3**				

Louis wants to give $15 to help kids who need school supplies. He also wants to buy a pair of shoes for $39. He gets $5 a week for his allowance. Louis remembers his sister's birthday is next month. He sets a goal of saving $16 for her gift. How many weeks does he have to save his allowance for all three of his goals?

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1.OA At the Park

Alignments to Content Standards: 1.OA.A.1

Task

a. There were 7 children at the park. Then 4 more showed up. How many children were at the park all together?

b. There were 7 children at the park. Some more showed up. Then there were 11 children in all. How many more children came?

c. There were some children at the park. Four more children showed up. Then there were 11 children at the park. How many children were at the park to start with?

IM Commentary

This task includes three different problem types using the "Add To" context with a discrete quantity; see "1.OA The Pet Snake" for an "Add To" problem with a continuous quantity. Table 1 in the glossary of the CCSSM offers a succinct overview of all addition and subtraction problem types.

Although students should experience and practice with all three problem types, they would not necessarily be introduced at the same time. Please see the K, Counting and Cardinality; K–5, Operations and Algebraic Thinking Progressions Document for in-depth information about issues related to students' learning of these kinds of problems.

While students are expected to add and subtract fluently within 10 in first grade (1.OA.6), they are not expected to add and subtract fluently within 20 until second grade (2.OA.2).

Edit this solution

Solution

Students may use objects, pictures, or equations to represent their solutions. The solutions show equations with a question mark representing the unknown value, but other symbols are often used. For example, 4 + ? = 11 might also be written 4 + ____ = 11 or 4 + ☐ = 11.

a. Total Unknown: There were 11 children in all.
Possible equation: $7+4=?$

b. Addend Unknown: 4 more children came.
Possible equation: $7+?=11$

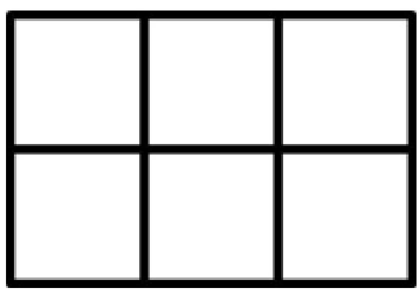
c. Start Unknown: There were 7 children in the park to start with.
Possible equation: $?+4=11$

3.MD, 3.G, 3.NF Halves, thirds, and sixths

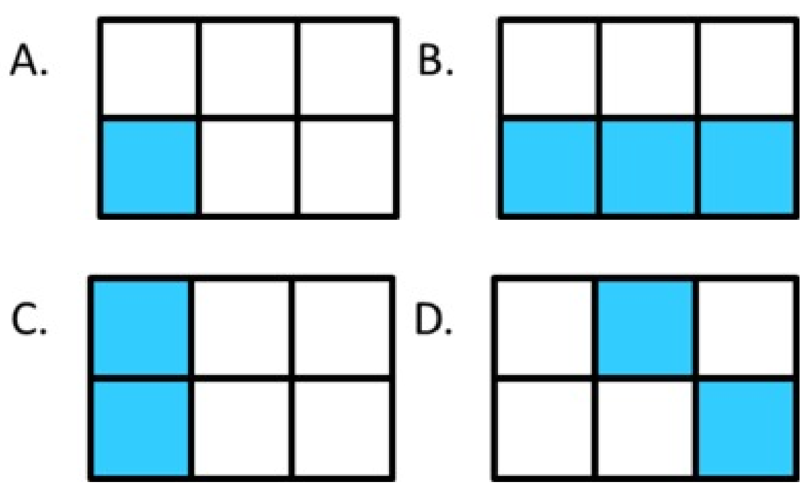
Alignments to Content Standards: 3.MD.C.6 3.G.A.2 3.NF.A.1 3.NF.A.3.b

Task

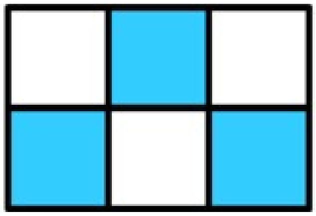
a. A small square is a square unit. What is the area of this rectangle? Explain.



b. What fraction of the area of each rectangle is shaded blue? Name the fraction in as many ways as you can. Explain your answers.



E.



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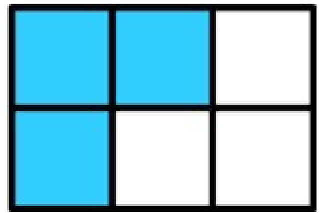

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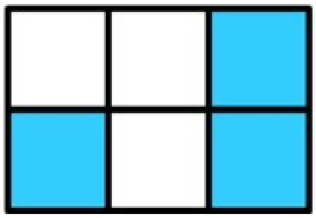
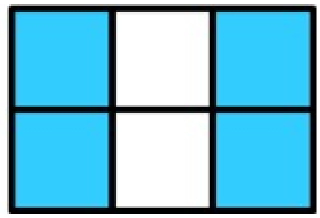
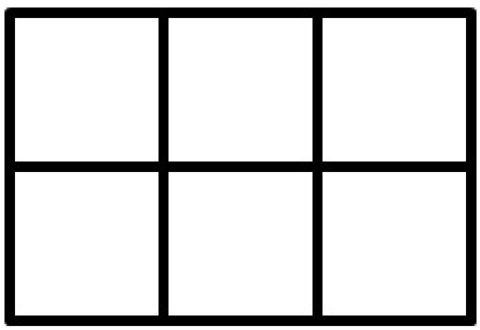


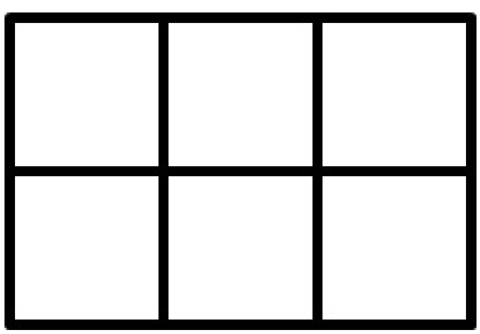
H.



c. Shade $\frac{1}{2}$ of the area of rectangle in a way that is different from the rectangles above.



d. Shade $\frac{2}{3}$ of the area of the rectangle in a way that is different from the rectangles above.



IM Commentary

The purpose of this task is for students to use their understanding of area as the number of square units that covers a region (3.MD.6), to recognize different ways of

6.NS Making Hot Cocoa, Variation 1

Task

One thermos of hot chocolate uses $\frac{2}{3}$ cup of cocoa powder. How many thermoses can Nelli make with 3 cups of cocoa powder?

a. Solve the problem by drawing a picture.

b. Explain how you can see the answer to the problem in your picture.

c. Which of the following multiplication or divisions equations represents this situation? Explain your reasoning.

$$3 \times \frac{2}{3} = ? \qquad 3 \div \frac{2}{3} = ? \qquad \frac{2}{3} \div 3 = ?$$

d. Solve the arithmetic problem you chose in part (c) and verify that you get the same answer as you did with your picture.

IM Commentary

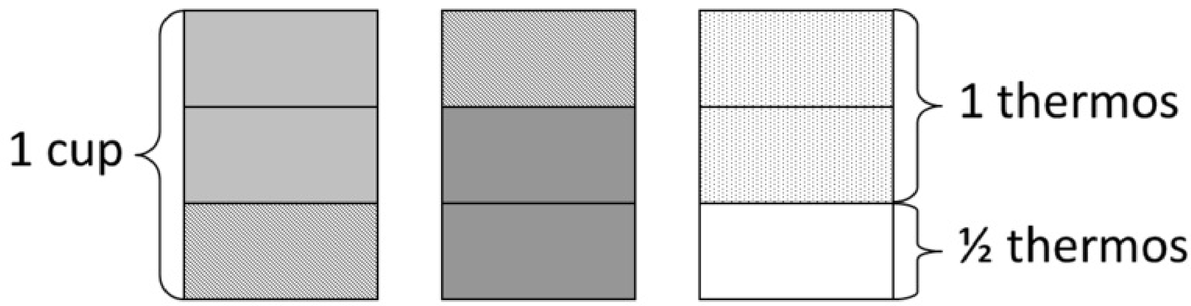
This is the first of two fraction division tasks that use similar contexts to highlight the difference between the “Number of Groups Unknown” a.k.a. “How many groups?” when the quotient is a fraction (or mixed number) greater than 1 (Variation 1) and when the quotient is a fraction that is less than 1 (Variation 2). Even when students understand this type of division context when they involve whole numbers, the transition to division problems involving fractions is not easy. In order to successfully make this transition, students must have a solid understanding of multiplication and division with whole numbers and multiplication with fractions.

These two tasks are meant as instructional tasks, with the idea that students who can solve these problems with diagrams may or may not see the connection to division. Thus, the tasks ask students to make this connection explicit.

There are significant language issues when moving from whole number to fraction division when the quotient is less than 1. In that case, it might help to point out that "Number of Groups Unknown" is better characterized as "Fraction of a Group Unknown" a.k.a. "What fraction of a group?" and the alternate of "Group Size Unknown" would be worded "How much in each group?"

Solution

a. Below is a picture that can be used to solve the problem.



b. The picture shows three rectangles that each represent 1 cup of cocoa powder. Each cup is divided into thirds. Since one thermos requires $\frac{2}{3}$ cup, 2 thirds are shaded to show a single thermos of cocoa. There are four whole groups of $\frac{2}{3}$ cups of cocoa and $\frac{1}{2}$ of a group of $\frac{2}{3}$ cups of cocoa shown in the picture

Nelli can make $4\frac{1}{2}$ thermoses of cocoa.

c. We have divided the 3 cups of cocoa powder into groups of size $\frac{2}{3}$, so we are finding out how many groups of $\frac{2}{3}$ there are in 3. So the correct equation is:

$$3 \div \frac{2}{3} = ?$$

d. Solve the arithmetic problem you chose in part (c) and verify that you get the same answer as you did with your picture.

$$\begin{aligned} 3 \div \frac{2}{3} &= \frac{3}{1} \times \frac{3}{2} \\ &= \frac{9}{2} \\ &= 4\frac{1}{2} \end{aligned}$$

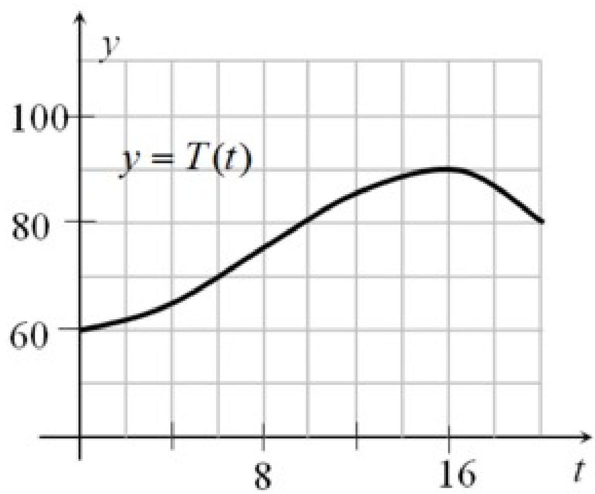
So the computation gives the same answer as we see in the picture.

F-IF Warming and Cooling

Alignments to Content Standards: F-IF.B.4

Task

The figure shows the graph of T , the temperature (in degrees Fahrenheit) over one particular 20-hour period in Santa Elena as a function of time t.



a. Estimate $T(14)$.

b. If $t = 0$ corresponds to midnight, interpret what we mean by $T(14)$ in words.

c. Estimate the highest temperature during this period from the graph.

d. When was the temperature decreasing?

e. If Anya wants to go for a two-hour hike and return before the temperature gets over 80 degrees, when should she leave?

IM Commentary

This task is meant to be a straight-forward assessment task of graph reading and interpreting skills. This task helps reinforce the idea that when a variable represents time, $t = 0$ is chosen as an arbitrary point in time and positive times are interpreted as times that happen after that.

Edit this solution

Solution

a. $T(14)$ is a little less than 90 degrees Fahrenheit; maybe 88 or 89 degrees.

b. The temperature was almost 90 degrees at 2:00 in the afternoon.

c. The highest temperature was about 90 degrees.

d. The temperature was decreasing between 4:00 p.m. and 8:00 p.m. It might have continued to decrease after that, but there is no information about the temperature after 8:00 p.m.

e. The temperature reaches 80 degrees just before 10:00 a.m. If Anya wants to go for a two-hour hike and return before the temperature gets over 80 degrees, then she should start her hike before 8:00 a.m.